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# Effect of Weightage of Jobs In Two Stage Flow Shop Scheduling To Minimize The Rental Cost With Jobs In A String Of Disjoint Job Blocks, Processing Time And Set Up Time Each Associated With Probabilities Including Breakdown Interval

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## Abstract

This paper studies two stage flow shop production scheduling problem to minimize the rental cost under the specified rental policy with weightage of jobs. The processing time and set up time are associated with probabilities. The concept of breakdown interval of machines is also included and the jobs are taken in a string of disjoint job blocks. Further the jobs are given weightages. The objective of the study is to find an algorithm to minimize the rental cost of machines under the specified rental policy with breakdown interval and a string of disjoint job blocks. Algorithm is justified by numerical illustration.

**Keywords**: Equivalent job, job weightage, Rental policy, Breakdown interval, String, Optimal sequence, Set up time.

## Introduction

In flow shop scheduling the objective is to obtain a sequence of jobs which when processed in a fixed order of machines, will optimize a well defined criteria. Various researchers have done a lot of work in this direction. Johnson(1954) gave a procedure to obtain an optimal sequence for n-jobs two-three machines flow shop scheduling problem with an objective of minimizing the makespan. The work was developed by Ignall & Scharge(1965) , Bagga (1969) , J.N.D Gupta(1975),Maggu&Dass(1977),Yoshida&Hitomi(1979),SinghT.P(1985),ChanderSekharan(1992),Anup(2002), Gupta Deepak(2005) etc by considering various parameters. Narain(2005) studied a problem to obtain a sequence which gives minimum possible rental cost while minimizing total elapsed time under pre defined rental policy. Singh T.P & Gupta Deepak(2006) studied two stage flow shop scheduling problem to minimize the rental cost by associating probabilities with the processing time. Gupta Deepak & Sameer Sharma(2012) studied two stage flow shop problem to minimize the rental cost by associating probabilities with processing time and set up time and further they used the concept of breakdown interval and job block. The study made by Gupta Deepak & Sameer Sharma(2012) is extended in the present paper by using the concept of a string of disjoint job blocks and the concept of weightage of jobs to solve further complex problems.

## Practical Situation

Various practical situations occur in real life when one gets the assignments but one does not have his own machines or does not have enough money to buy the machines.Under such conditions one can take the machines on rent in order to complete the assignments.Also this paper is very important for those fields where priority of one job over another is considered.Also some time,machines are not available due to any reason.So the concept of breakdown interval is also included. The temporal lack of machine availability is known as breakdown(due to failure of electricity,lack of raw materials etc).Before 1954,the concept of breakdown had not considered by any author.In 1954,Johnson considered the effect of breakdown interval..Also during the production process,a foreman takes a certain time to set up a particular machine for the processing of a particular job and that particular time is called set up time.So that is why the concept of set up time is also taken in an account.The concept of weightages of jobs is also included.Weightage given to a particular job shows its relative importance

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among all the jobs in the group.A weightage factor is assigned to each job,Which will denote its importance among all the jobs during the production process.

## Notations

S : sequence of jobs1,2,3…….
$M_j$: machine j, j=1,2,
$A_i$ : processing time of ith job on machine A
$B_i$: processing time of ith job on machine B
$A_i'$ : expected processing time of ith job on machine A
$B_i'$ : expected processing time of ith job on machine B
$p_i$ : probability associated with the processing time $A_i$ of ith job on machine A
$q_i$ : probability associated with the processing time $B_i$ of ith job on machine B
$S_i^A$ : set up time of ith job on machine A
$S_i^B$ : set up time of ith job on machine B
$r_i$ : probability associated with $S^A{}_i$
$s_i$ : probability associated with $S^B{}_i$
$A_i''$ : expected processing time of ith job on machine A after considering the effect of breakdown interval
$B_i''$ : expected processing time of ith job on machine B after considering the effect of breakdown interval
$S'$ : optimal string before considering the effect of breakdown interval
$S''$ : optimal string after considering the effect of breakdown interval
$c_j$ : rental cost per unit time of machine j.

## Assumptions

We assume that all the machines are taken on rent as and when they are required and are returned as when they are no longer required.Under this policy second machine is taken on rent at a time when first job completes its processing on first machine.

1) Jobs are independent to each other.
2) Machine breakdown interval is deterministic,i.e. the breakdown interval is well known in advance.
3) Once a job started on a machine ,the process on that machine can not be stopped unless the job is completed.

## Rental Policy

Machines are taken on rent as and when they are required and are returned as and when they are no longer required,i.e first machine will be taken on rent in the starting of the processing of the jobs and second machine will be taken on rent after the completion of first job on ist machine.

## Problem Formulation

Let n jobs are to be processed on two machines A and B under the specified rental policy.Let $A_i$ be the processing time of $i^{th}$ job on $1^{st}$ machine with probabilities $p_i$ and $s_i^A$ be the set up time of $i^{th}$ job on $1^{st}$ machine with probabilities $r_i$.Similarly for machine B.Our aim is to find a sequence $\{S_k\}$ of jobs so as to minimize the rental cost.

| Jobs | Machine A | | | | Machine B | | | | Weights |
|---|---|---|---|---|---|---|---|---|---|
| i | $A_i$ | $p_i$ | $S_i^A$ | $r_i$ | Bi | $q_i$ | $S_i^B$ | $s_i$ | $W_i$ |
| 1 | $A_1$ | $p_1$ | $S_1^A$ | $R_1$ | $B_1$ | $q_1$ | $S_1^B$ | $S_1$ | $W_1$ |
| 2 | $A_2$ | $p_2$ | $S_2^A$ | $R_2$ | $B_2$ | $q_2$ | $S_2^B$ | $S_2$ | $W_2$ |
| 3 | $A_3$ | $p_3$ | $S_3^A$ | $R_3$ | $B_3$ | $q_3$ | $S_3^B$ | $S_3$ | $W_3$ |
| 4 | $A_4$ | $p_4$ | $S_4^A$ | $R_4$ | $B_4$ | $q_4$ | $S_4^B$ | $S_4$ | $W_4$ |
| 5 | $A_5$ | $p_5$ | $S_5^A$ | $R_5$ | $B_5$ | $q_5$ | $S_5^B$ | $S_5$ | $W_5$ |

## Algorithm

**STEP 1**: Define expected processing time $A_i'$& $B_i'$ on machines A and B as follows
$A_i' = A_i \times p_i$ and $B_i' = B_i \times q_i$ and solve further by considering the effect of given weights of jobs as follows,

Compute $A_{i1}'$ and $A_{i2}'$ as follows:
1, if min $(A_{ij}) = A_{i1}$ for j= 1,2
Then $A_{i1}' = A_{i1} + W_i$ and $A_{i2}' = A_{i2}$
2, if min $(A_{ij}) = A_{i2}$ for j = 1,2.
Then $A_{i1}' = A_{i1}$ and $A_{i2}' = A_{i2} + W_i$
Now find $A_{ij} = A_{ij} / W_i$
i = 1,2,,,,,,,,,,,,,,,n and j = 1,2.

**STEP 2**: Arrange order of jobs in an optimal way in the job block β by using Johnson technique.Let it be β'.
**STEP 3**: Define expected processing time of job blocks ϒ and β' by using technique of Maggu&Dass.
**STEP 4**: Using Johnson´s two machine algorithm, obtain a sequence S while minimizing total elapsed time.
**STEP 5**: Prepare a flow time table for the sequence obtained in step 4 and read effect of breakdown interval (a,b) on different jobs.

**STEP 6**: Form a reduced problem with processing times $A_i''$ and $B_i''$
If breakdown interval has effect on job i then,

$A_i'' = A_i' + L$
and $B_i'' = B_i' + L$ ,
where L=b-a(length of interval).

If breakdown interval has no effect on job i then,

$A_i'' = A_i'$
and $B_i'' = B_i'$

**STEP 7**: Again arrange order of jobs in an optimal way in job block β by using Johnson technique.Let it be K.
**STEP 8**: Define expected processing time of job blocks ϒ and K by using technique of Maggu and Dass.
**STEP 9**: Now repeat the procedure to get optimal sequence S as in step 4.
**STEP 10**: Observe the processing time of first job of above sequence and let it be P.
**STEP 11**: Obtain all the jobs having processing times on A greater than P.Put these jobs one by one in the first position of above optimal sequence S and take the same order of the remaining jobs in every case.Let these sequences be $S_1, S_2, S_3, \ldots S_r$.
**STEP 12**: Prepare in-out flow tables for all above sequences(only those which contain job block ϒ and the string K.Evaluate total completion time of last job of each sequence i.e, $t_1(S_i)$ & $t_2(S_i)$ on each machine A and B respectively.
**STEP 13**: Evaluate completion time $CT(S_i)$ of first job of each above selected sequence on machine A.
**STEP 14**: Calculate utilization time $U_i$ of second machine for each of above sequence as $U_i = t_2(S_i) - CT(S_i)$ for i=1,2,3,…..
**STEP 15**: Find min$\{U_i\}$, i=1,2,3…..,let it be corresponding to i=m,then $S_m$ is the optimal sequence for minimizing rental cost,

Min rental cost = $\{t_1(s_m)\times C_1\} + \{U_m \times C_2\}$ ,
where $C_1$ = Rental cost of first machine per unit time.
$C_2$ = Rental cost of second machine per unit time.

## Numerical

Consider 5 jobs 2-machine problem to minimize rental cost. Processing time and set up time each associated with probabilities. Let a string S=(α,β) ,where α=(1,4) & β=(2,3,5).Also α is fixed and β is an arbitrary sequence. Respective weightage of jobs are given. Rental cost per unit time of machine A is 10 and for machine B is 20.Find minimum rental cost.

| Jobs | Machine A | | | | Machine B | | | | |
|---|---|---|---|---|---|---|---|---|---|
| i | $A_i$ | $p_i$ | $S_i^A$ | $r_i$ | $B_i$ | $q_i$ | $S_i^B$ | $s_i$ | $W_i$ |
| 1 | 6 | 0.2 | 2 | 0.2 | 4 | 0.2 | 3 | 0.1 | 1 |
| 2 | 6 | 0.2 | 3 | 0.3 | 6 | 0.2 | 1 | 0.2 | 1 |
| 3 | 4 | 0.1 | 1 | 0.1 | 3 | 0.3 | 1 | 0.2 | 2 |
| 4 | 5 | 0.3 | 2 | 0.2 | 5 | 0.2 | 3 | 0.4 | 3 |
| 5 | 8 | 0.2 | 2 | 0.2 | 6 | 0.1 | 4 | 0.1 | 2 |

## Solution

**Step 1:** Find expected processing time,

| Jobs | $A_i'$ | $B_i'$ | $W_i$ |
|---|---|---|---|
| 1 | 0.9 | 0.4 | 1 |
| 2 | 1.0 | 0.3 | 1 |
| 3 | 0.2 | 0.8 | 2 |
| 4 | 0.3 | 0.6 | 3 |
| 5 | 1.2 | 0.2 | 2 |

Now considering the effect of given weightages of jobs,

| Jobs | $A_i'$ | $B_i'$ |
|---|---|---|
| 1 | 0.9 | 1.4 |
| 2 | 1.0 | 1.3 |
| 3 | 1.1 | 0.4 |
| 4 | 1.1 | 0.2 |
| 5 | 0.6 | 1.1 |

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**Step 2:** Now arrange the order of jobs 0f β in anoptimal way by using Johnson technique,

| Jobs | $A_i'$ | $B_i'$ |
|---|---|---|
| 2 | 1.0 | 1.3 |
| 3 | 1.1 | 0.4 |
| 5 | 0.6 | 1.1 |

So the optimal sequence is $\beta' = (5,2,3)$

**Step 3:** Now we find $A_\alpha'$ , $B_\alpha'$ , $A_{\beta'}'$ , $B_{\beta'}'$ by maggu & Das

$$A_\alpha' = A_1' + A_4' - \min(A_4', B_1')$$
$$= 0.9 + 1.1 - 1.1 = 0.9$$
$$B_\alpha' = B_1' + B_4' - \min(A_4', B_1')$$
$$= 1.4 + 0.2 - 1.1 = 0.5$$

Now to find $A_{\beta'}'$ and $B_{\beta'}'$

Let $\gamma = (5,2)$ so that $\beta' = (\gamma, 3)$

$$\text{Now } A_\gamma' = A_5' + A_2' - \min(B_5', A_2')$$
$$=0.6 + 1.0 - 1.0 = 0.6$$
$$B_\gamma' = B_5' + B_2' - \min(B_5', A_2')$$
$$= 1.1 + 1.3 - 1.0 = 1.4$$
$$\text{So } A_{\beta'}' = A_\gamma' + A_3' - \min(B_\gamma', A_3')$$
$$= 0.6 + 1.1 - 0.2 = 1.5$$
$$B_{\beta'}' = B_\gamma' + B_3' - \min(B_\gamma', A_3')$$
$$= 1.4 + 0.4 - 0.2 = 1.6$$

**Step 4:** The reduced problem is

| Jobs | Machine A | Machine B |
|---|---|---|
| Α | 0.9 | 0.5 |
| $\beta'$ | 1.5 | 1.6 |

Now using Johnson technique,optimal string is

$$S' = (\beta', \alpha) = (5,2,3,1,4)$$

**Step 5:** In-out table for above sequence

| Jobs | Machine A | Machine B |
|---|---|---|
| 5 | 0.0 – 1.6 | 1.6 – 2.2 |
| 2 | 2.0 – 3.2 | 3.2 – 4.4 |
| 3 | 4.1 – 4.5 | 4.6 – 5.5 |
| 1 | 4.6 – 5.8 | 5.8 – 6.6 |
| 4 | 6.2 – 7.7 | 7.7 – 8.7 |

**Step 6:** Now consider the effect of break down interval(5,10)

| Jobs | Machine A | Machine B |
|---|---|---|
| 1 | 5.9 | 6.4 |
| 2 | 1.0 | 1.3 |
| 3 | 1.1 | 5.4 |
| 4 | 6.1 | 5.2 |
| 5 | 0.6 | 1.1 |

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**Step 7:** Now we arrange the order of jobs in β in an optimal way

| Jobs | Machine A | Machine B |
|---|---|---|
| 2 | 1.0 | 1.3 |
| 3 | 1.1 | 5.4 |
| 5 | 0.6 | 1.1 |

Optimal sequence is K = (5,2,3)

**Step 8:** Find $A_\alpha''$ and $B_\alpha''$, $A_K''$, $B_K''$.

$$A_\alpha'' = A_1'' + A_4'' - \min(A_4'', B_1'')$$
$$= 5.9 + 6.1 - 6.1 = 5.9$$
$$B_\alpha'' = B_1'' + B_4'' - \min(A_4'', B_1'')$$
$$= 6.4 + 5.2 - 6.1 = 5.5$$

Now to find $A_K''$ and $B_K''$

Let γ = (5,2) so that K=(γ,3)

$$\text{Now } A_\gamma'' = A_5'' + A_2'' - \min(A_2'', B_5'')$$
$$= 0.6 + 1.0 - 1.0 = 0.6$$
$$B_\gamma'' = B_5'' + B_2'' - \min(A_2'', B_5'')$$
$$= 1.1 + 1.3 - 1.0 = 1.4$$
$$\text{So } A_K'' = A_\gamma'' + A_3'' - \min(A_3'', B_\gamma'')$$
$$= 0.6 + 1.1 - 1.1 = 0.6$$
$$B_K'' = B_\gamma'' + B_3'' - \min(A_3'', B_\gamma'')$$
$$= 1.4 + 5.4 - 1.1 = 5.7$$

**Step 9:** The reduced problem is

| Jobs | $A_i''$ | $B_i''$ |
|---|---|---|
| A | 5.9 | 5.5 |
| K | 0.6 | 5.7 |

Optimal string $S''$ = (K,α) = (5,2,3,1,4)

**Step 10:** In-out table for above sequence $S''$ is as follows

| Jobs | Machine A | Machine B |
|---|---|---|
| 5 | 0.0 – 1.6 | 1.6 – 2.2 |
| 2 | 2.0 – 3.2 | 3.2 – 4.4 |
| 3 | 4.1 – 4.5 | 4.6 – 5.5 |
| 1 | 4.6 – 5.8 | 5.8 – 6.6 |
| 4 | 6.2 – 7.7 | 7.7 – 8.7 |

Total time on machine A = 7.7
Total time on machine B = 8.7 – 1.6 =7.1
Total rental cost = (7.7×10) + (7.1×20) = 219

**Step 11:** Now we see that the processing time of $5^{th}$ job i.e, the very first job of optimal sequence $S''$ = (5,2,3,1,4) on first machine is 0.6.

So other optimal sequences are given below,

$S_1$ = (1,4,5,2,3)
$S_2$ = (2,3,1,4,5)
$S_3$ = (3,1,4,5,2)
$S_4$ = (4,5,2,3,1)

But from these sequences only $S_1$ satisfies conditions of job blocks α and K , So we reject all other sequences.

**Step 12:** Now in-out table for $S_1$ = (1,4,5,2,3)

| Jobs | Machine A | Machine B |
|---|---|---|
| 1 | 0.0 – 1.2 | 1.2 – 2.0 |
| 4 | 1.6 – 3.1 | 3.1 – 4.1 |
| 5 | 3.5 – 5.1 | 5.3 – 5.9 |
| 2 | 5.5 – 6.7 | 6.7 – 7.9 |
| 3 | 7.6 – 8.0 | 8.1 – 9.0 |

Total time on machine A = 8.0
Total time on machine B = 9.0 – 1.2 = 7.8
Total rental cost = (8.0×10) + (7.8×20) = 236

Clearly $S''$ is the optimal sequence which gives the minimum total rental cost and obeys job block restrictions,So minimum total rental cost is 219.

## Conclusion

So we conclude that the complex problems of scheduling containing a number of variables are also solvable by the well defided techniques as above.

## Remark

The study can be extended by using the concept of Transportation time.

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